Relativistic Dynamics of Vector Bosons in the Field of Gravitational Radiation

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We consider a model of the state evolution of relativistic vector bosons, which includes both the dynamical equations for the particle four-velocity and the equations for the polarization four-vector evolution in the field of a nonlinear plane gravitational wave. In addition to the gravitational minimal coupling, tidal forces linear in curvature tensor are suggested to drive the particle state evolution. The exact solutions of the evolutionary equations are obtained. Birefringence and tidal deviations from the geodesic motion are discussed.

1 INTRODUCTION

The formulation of the covariant dynamic equations for the particles with internal structure or with supplementary degrees of freedom unavoidably involves consideration the *tidal* forces, i. e., forces linear in the curvature tensor.

A. Papapetrou in the paper(1), deriving the covariant equations for the spinning particle, presented the first example of the tidal forces. Considering the multipole representation of the interaction between the particle possessing internal structure and the external field, W. G. Dixon(2) and A. H. Taub(3) introduced the coupling of the curvature and the particle quadrupole moment. W. Israel(4) formulated the covariant dynamics of the macroscopic polarization in the medium also using the tidal interactions. At present the covariant theory of the dynamics of spinning objects is well developed due to the interest to the problem of emission of the gravitational waves (e.g., Ref. 5).

As it was shown by I. T. Drummond and S. J. Hathrell in Ref. 6, the tidal terms appear in the covariant electrodynamics. The birefringence induced by curvature(6−8) is an electrodynamical phenomenon, which also admits interpretation in terms of the photon motion under the influence of tidal forces. This interpretation is based on the fact that due to gravitationally induced birefringence the energy and the instant direction of the photon momentum vector depend on the direction of the polarization vector, rotating in the gravity field.

On the other hand, when the electromagnetic fields dominate and the tidal forces are negligible in comparison with them, we can use the relativistic Bargmann-Michel-Telegdi equations(9), describing the minimal interaction with gravitation and do not displaying the

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tidal forces. Nevertheless, as it have been emphasized by I. B. Khriplovich[10], there exists a direct analogy between electromagnetic and tidal coupling of the particle spin. One can introduce, as was advocated by Khriplovich[10], new part of the Hamiltonian, namely, $-\frac{1}{4}R_{abcd}S^{ab}S^{cd}$, where R_{abcd} is a Riemann curvature tensor and S^{ab} is a spin tensor, instead of (or in addition to) the electromagnetic coupling part $\frac{e}{c}F_{ab}S^{ab}$. In other words, both electromagnetic and tidal coupling of spin can be described analogously. Finally, I. B. Khriplovich noted in Ref. 10 that the developed formalism and the corresponding equations are applicable to the vector bosons with the spin equal to one.

We consider this idea as an initial point for our investigations and we use the evolutionary equations for vector boson with vanishing electromagnetic field and dominating tidal forces (see the Section 2 of this paper). In the Section 3 we integrate the dynamic equations exactly for the case of nonlinear gravitational wave background and present two particular simplified exact toy-models.

2 EVOLUTIONARY EQUATIONS

The covariant model of the vector boson evolution contains the pair of equations.

$$\frac{DU^i}{D\tau} = -\frac{1}{mc}R^{*i}_{\cdot klm}U^k\Xi^l U^m, \tag{1}$$

$$\frac{D\Xi^i}{D\tau} = -\frac{1}{mc}R^{*i}_{\cdot klm}\Xi^k\Xi^l U^m. \tag{2}$$

The first equation is a dynamical one for the determination of the particle four-velocity time-like vector $U^i \equiv \frac{dx^i}{d\tau}$ ($U^iU_i = 1$). The second equation is the equation of evolution of the polarization space-like four-vector Ξ^i ($\Xi^i\Xi_i = -E^2$, the constant E has the dimensionality of the Planck constant). D denotes the covariant differential, τ is the affine parameter along the particle world-line (particle proper time). The tensor $R^{*i}_{\cdot klm}$ is a right-dual to Riemann tensor

$$R^{*i}_{\cdot klm} \equiv \frac{1}{2}R^i_{\cdot kpq}\epsilon^{pq}_{\cdot\cdot lm}. \tag{3}$$

The Levi-Civita tensor $\epsilon^{pq}_{\cdot\cdot lm}$ is defined as usual with

$$\epsilon^{pqlm} = \frac{1}{\sqrt{-g}}\ E^{pqlm}, \quad \epsilon_{pqlm} = \sqrt{-g}\ E_{pqlm}, \tag{4}$$

where E^{pqlm} is a completely skew-symmetric symbol with $E^{0123} = -E_{0123} = 1$.

The equation (1) looks like the relativistic dynamic equation $mc\frac{DU^i}{D\tau} = F^i$ with the force four-vector F^i expressed in terms of the Riemann tensor. It is a typical "tidal force". This force is evidently orthogonal to the four-velocity vector ($F^iU_i = 0$), and this fact combined with the velocity normalization law admits us to speak about particle mass conservation.

Analogously, one can consider the equation (2) as an equation with "tidal force" acting on the polarization four-vector. The right part of this equation is apparently orthogonal to the Ξ^i vector. Consequently, $\Xi^k\Xi_k = -E^2$ is a constant of motion.

Finally, one can see that due to (1) and (2)

$$\frac{D}{D\tau}(U^i\Xi_i) = U_i\frac{D\Xi^i}{D\tau} + \Xi_i\frac{DU^i}{D\tau} \equiv 0, \tag{5}$$

i.e. the scalar product $(U^i\Xi_i) = const$ is an integral of motion. This constant is considered to be equal to zero in order to display the law of orthogonality of the four-velocity vector and polarization four-vector for the massive vector boson as well as for the massless photon[(10)].

One can note that S^i and Ξ^i quantities are considered to be pseudo-vectors, nevertheless, the product of the right-dual Riemann tensor with spin or polarization is a true tensor. For the sake of simplicity we shall omit the prefix "pseudo" below.

2.1 Remark on the derivation of the evolutionary equations

The simplest way to obtain the evolutionary equations (1), (2) is the following. Let us consider the Bargmann-Michel-Telegdi equations[(9)] with gyromagnetic ratio $g = 2$, i.e. in the case of the absence of the anomalus magnetic moment:

$$\frac{DU^i}{D\tau} = \frac{e}{mc^2}F^i_{\cdot k}\,U^k, \quad \frac{DS^i}{D\tau} = \frac{e}{mc^2}F^i_{\cdot k}\,S^k. \tag{6}$$

Then, by the analogy, discussed by I. B. Khriplovich[(10)], let us replace the term describing the contribution of the electromagnetic field $\frac{e}{mc^2}F^i_{\cdot k}$, by the tidal contribution $-\frac{1}{2mc}R^i_{\cdot klm}S^{lm}$. If we express the spin-tensor S^{lm} in terms of spin vector

$$S^{lm} = \epsilon^{lmpq}U_pS_q, \quad S_i = \frac{1}{2}\epsilon_{iklm}U^kS^{lm}, \tag{7}$$

use the definition of the right-dual Riemann tensor (3), and then replace the spin four-vector S^i by the polarization four-vector Ξ^i, we shall obtain the equations (1) and (2).

2.2 Gravitational wave background

The model of the plane gravitational wave (GW) background based on the exact solution of the Einstein equations in vacuum[(11)] is well-known and highly fruitful. The metric of the plane GW can be represented in the following form:

$$ds^2 = 2dudv - L(u)^2\{[e^{2\beta(u)}(dx^2)^2 + e^{-2\beta(u)}(dx^3)^2]\cosh 2\gamma(u) + 2\sinh 2\gamma(u)dx^2dx^3\}, \tag{8}$$

where

$$u \equiv (ct - x^1)/\sqrt{2}, \qquad v \equiv (ct + x^1)/\sqrt{2} \tag{9}$$

are the retarded and the advanced time, respectively. The functions $L(u), \beta(u), \gamma(u)$ are coupled by one equation only[11]

$$L''(u) + L(u)\left\{[\beta'(u)]^2 \cosh^2 \gamma(u) + [\gamma'(u)]^2\right\} = 0, \tag{10}$$

which is the unique remaining nontrivial Einstein's equation for the case of vacuum. The prime denotes the derivative by the retarded time u. The functions $\beta(u)$ and $\gamma(u)$ are assumed to be arbitrary ones. We shall say that the gravitational wave is of the first polarization, if $\beta(u) \neq 0$, $\gamma(u) = 0$, and of the second polarization, if $\beta(u) = 0$, $\gamma(u) \neq 0$.

The initial conditions for the functions $L(u), \beta(u), \gamma(u)$ are the following:

$$L(0) = 1, \quad L'(0) = 0, \quad \beta(0) = \gamma(0) = 0, \quad \beta'(0) = \gamma'(0) = 0. \tag{11}$$

The Riemann tensor calculated with the metric (8) has only four nontrivial components

$$-R^2_{\cdot u2u} = R^3_{\cdot u3u} = L^{-2}[L^2 \beta' \cosh^2 2\gamma]',$$

$$R^3_{\cdot u2u} = -L^{-2}[L^2 e^{2\beta}(\gamma' - \beta' \sinh 2\gamma \cosh 2\gamma)]',$$

$$R^2_{\cdot u3u} = -L^{-2}[L^2 e^{-2\beta}(\gamma' + \beta' \sinh 2\gamma \cosh 2\gamma)]'. \tag{12}$$

The components of the right-dual Riemann tensor are equal, correspondingly, to

$$R^*_{2u2u} = -L^2 \cdot R^3_{\cdot u2u}, \quad R^*_{3u3u} = -L^2 \cdot R^2_{\cdot u3u},$$

$$R^*_{2u3u} = R^*_{3u2u} = L^2 \cdot R^2_{\cdot u2u} = -L^2 \cdot R^3_{\cdot u3u}\ . \tag{13}$$

The space-time with metric (8) admits the existence of five Killing's vectors, three of them, namely,

$$\xi^i_{(v)} = \delta^i_v, \quad \xi^i_{(2)} = \delta^i_2, \quad \xi^i_{(3)} = \delta^i_3, \tag{14}$$

form the Abelian subgroup. The vector $\xi^i_{(v)}$ possesses the following properties:

$$g_{ik}\xi^i_{(v)}\xi^k_{(v)} = 0, \quad \nabla_k \xi^i_{(v)} = 0, \quad \xi^i_{(v)} R_{iklm} = 0 = \xi^i_{(v)} R^*_{iklm}, \tag{15}$$

i.e., it is a null covariantly constant vector, orthogonal to the Riemann tensor and to its dual tensors.

3 THE EXACT SOLUTION FOR THE MODEL DYNAMICAL SYSTEM

3.1 The first integrals of motion

We have found (see Section 2) that due to the structure of the evolutionary equations three quadratic quantities

$$U^i U_i = const \equiv 1, \quad \Xi_i \Xi^i = const \equiv -E^2, \quad \Xi_i U^i = const \equiv 0 \tag{16}$$

are constant along the particle world-line independently on the gravitational background properties.

Next two integrals of motion

$$U_v \equiv= g_{ik}\ \xi^i_{(v)} U^k = const \equiv C_v, \quad \Xi_v \equiv= g_{ik}\ \xi^i_{(v)} \Xi^k = const \equiv E_v \tag{17}$$

exist due to the GW symmetry. The convolution of the equations (1) and (2) with $g_{ik}\xi^k_{(v)}$ gives the formulae (17), if we use the properties (15) of the null covariantly constant Killing vector.

Finally, accounting that

$$\frac{du}{d\tau} = U^u = U_v = C_v, \tag{18}$$

we can link the particle proper time τ with the retarded time u:

$$\tau = \frac{u}{C_v} + const. \tag{19}$$

Note, that for massive boson $C_v \neq 0$. We shall use the relationship (19) with the constant equal to zero in order to reparametrize the remaining differential equations for the components of U^i and Ξ^i, parallel to the GW front plane.

3.2 The motion in the GW front plane

Let us extract four equations from (1) and (2) by means of convolution of (1),(2) with the second and the third Killing vectors from the abelian subgroup (14). Denoting the corresponding convolutions by

$$U_\alpha \equiv g_{ik}\ \xi^i_{(\alpha)} U^k, \quad \Xi_\alpha \equiv g_{ik}\ \xi^i_{(\alpha)} \Xi^k, \tag{20}$$

where the Greek indices run from 2 to 3, we obtain the following system of coupled equations:

$$U'_\alpha(u) = -\frac{1}{mc}\ R^*_{\alpha u \gamma u} g^{\gamma\beta} [C_v \Xi_\beta(u) - E_v U_\beta(u)], \tag{21}$$

$$\Xi'_\alpha(u) = \frac{1}{C_v}\left[\frac{1}{2} g'_{\alpha\gamma}(u) - \frac{E_v}{mc}\ R^*_{\alpha u \gamma u}\right] g^{\gamma\beta}[C_v \Xi_\beta(u) - E_v U_\beta(u)]. \tag{22}$$

The evident symmetry of the equations (21),(22) allows us to introduce some new unknown functions

$$X_\alpha(u) \equiv C_v \Xi_\alpha(u) - E_v U_\alpha(u). \tag{23}$$

Then we obtain the following two-dimensional *key subsystem* as the differential consequence of (21),(22):

$$\begin{pmatrix} X'_2 \\ X'_3 \end{pmatrix} = \begin{pmatrix} a^2_2 & a^3_2 \\ a^2_3 & a^3_3 \end{pmatrix} \begin{pmatrix} X_2 \\ X_3 \end{pmatrix}. \tag{24}$$

Here

$$a_2^2(u) = \frac{L'}{L} + \beta' \cosh^2 2\gamma, \quad a_2^3(u) = e^{2\beta}[\gamma' - \beta' \sinh 2\gamma \cosh 2\gamma],$$

$$a_3^2(u) = e^{-2\beta}[\gamma' + \beta' \sinh 2\gamma \cosh 2\gamma], \quad a_3^3(u) = \frac{L'}{L} - \beta' \cosh^2 2\gamma. \tag{25}$$

It is interesting to mention that the key subsystem happens to be self-closed and does not contain the tidal terms with the Riemann tensor. Such two-dimensional subsystems were considered, solved and used in Refs. 8 and 12. The solution of (24), (25) is the following

$$\begin{pmatrix} X_2 \\ X_3 \end{pmatrix} = L \begin{pmatrix} e^{\beta} & 0 \\ 0 & e^{-\beta} \end{pmatrix} \begin{pmatrix} \cosh\gamma & \sinh\gamma \\ \sinh\gamma & \cosh\gamma \end{pmatrix} \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} X_2(0) \\ X_3(0) \end{pmatrix}, \tag{26}$$

where the function $\psi(u)$ has the form

$$\psi(u) \equiv \int\limits_0^u \beta' \sinh 2\gamma du. \tag{27}$$

The initial data $X_\alpha(0)$ are predetermined by those for the four-velocity and polarization vectors:

$$X_\alpha(0) = E_\alpha C_v - E_v C_\alpha, \quad C_\alpha \equiv U_\alpha(0), \quad E_\alpha \equiv \Xi_\alpha(0). \tag{28}$$

Then the solution of the total four-dimensional subsystem of equations (21),(22) can be reduced to the quadratures

$$\Xi_\alpha(u) = \frac{1}{C_v}\left[X_\alpha(u) + E_v U_\alpha(u)\right], \tag{29}$$

$$U_\alpha(u) = C_\alpha - \frac{1}{mc}\int\limits_0^u du \; R^*_{\alpha u \gamma u}(u) g^{\gamma\beta}(u) X_\beta(u). \tag{30}$$

In (8),(12),(13) and (26) one can find all the functions which are necessary for the integration in (30).

3.3 The quadratures for the remaining unknown functions

Now, when X_α are obtained and $U_\alpha(u)$ as well as $\Xi_\alpha(u)$ are represented via X_α, we can extract U_u and X_u from the quadratic integrals (16):

$$U_u(u) = \frac{1}{2C_v}[1 - g^{\alpha\sigma}(u) U_\alpha(u) U_\sigma(u)], \qquad C_v \neq 0, \tag{31}$$

$$\Xi_u(u) = -\frac{1}{2E_v}[E^2 + g^{\alpha\sigma}(u)\Xi_\alpha(u)\Xi_\sigma(u)], \qquad E_v \neq 0. \tag{32}$$

Finally, the particle position as a function of the retarded time is described by the formulae

$$x^\alpha(u) = x^\alpha(0) + \frac{1}{C_v}\int\limits_0^u g^{\alpha\beta}(u) U_\beta(u) du, \tag{33}$$

$$v(u) = v(0) + \frac{1}{C_v}\int\limits_0^u U_u(u)du. \tag{34}$$

Thus, the formulae (17),(19),(29)-(34) represent in the quadratures the *exact solution* of the equations (1),(2), containing 12 functions of the proper time τ.

3.4 The first exact toy-model

If the massive boson was at rest at the initial moment $u = 0$ in the chosen reference frame, i.e.

$$C_2 = C_3 = 0, \quad C_v = \frac{1}{\sqrt{2}} = U_u(0), \tag{35}$$

and if the GW has only the first polarization, i.e. $\gamma(u) = 0$, then $\psi(u) \equiv 0$ and we can simplify the formulae (26),(30),(29):

$$X_2(u) = \frac{1}{\sqrt{2}}Le^{\beta}E_2, \quad X_3(u) = \frac{1}{\sqrt{2}}Le^{-\beta}E_3, \quad g^{\alpha\beta}X_\alpha X_\beta = -\frac{1}{2}(E_2^2 + E_3^2), \tag{36}$$

$$U_2(u) = -\frac{E_3}{\sqrt{2}mc}\left(Le^{\beta}\right)', \quad U_3(u) = \frac{E_2}{\sqrt{2}mc}\left(Le^{-\beta}\right)', \tag{37}$$

$$\Xi_2(u) = Le^{\beta}E_2 - \frac{E_vE_3}{mc}\left(Le^{\beta}\right)', \quad \Xi_3(u) = Le^{-\beta}E_3 + \frac{E_vE_2}{mc}\left(Le^{-\beta}\right)'. \tag{38}$$

The relationships (16) yield

$$2E_v^2 + E_2^2 + E_3^2 = E^2, \tag{39}$$

i.e. one of the constants E_v, E_2 or E_3 can be expressed in terms of other ones and of the E normalization constant.

Computing the particle energy

$$\mathcal{E}(u) \equiv mc^2U_0 = \frac{mc^2}{\sqrt{2}}(U_u + U_v) = \frac{mc^2}{2\sqrt{2}C_v}(1 - g^{\alpha\beta}U_\alpha U_\beta + 2C_v^2), \tag{40}$$

and using (37) we obtain

$$\mathcal{E}(u) - mc^2 = \frac{1}{4m}\left[E_3^2\left(\frac{L'}{L} + \beta'\right)^2 + E_2^2\left(\frac{L'}{L} - \beta'\right)^2\right]. \tag{41}$$

Finally, let us calculate the U_1 component of the four-velocity vector:

$$U_1(u) \equiv \frac{1}{\sqrt{2}}(U_v - U_u) = -\frac{1}{4m^2c^2}\left[E_3^2\left(\frac{L'}{L} + \beta'\right)^2 + E_2^2\left(\frac{L'}{L} - \beta'\right)^2\right]. \tag{42}$$

3.5 The second toy-model

Let the GW be of the first polarization as well. Let at the moment $u = 0$ the polarization three-vector of a particle be directed along the $0x^1$ axis, and the initial particle motion be one-dimensional, for example along $0x^2$ axis. It is possible, if the constants are chosen as follows:

$$E_2 = E_3 = 0, \quad E_v = -\Xi_u(0) = \frac{E}{\sqrt{2}}, \quad C_3 = 0, \quad C_v = U_u(0) = \sqrt{\frac{1 + C_2^2}{2}}. \tag{43}$$

Then we see that

$$U_2(u) = C_2, \quad U_3(u) = -\frac{C_2 E}{\sqrt{2}mc} \left(L e^{-\beta}\right)', \tag{44}$$

$$U_1(u) = \frac{C_2^2}{2\sqrt{1 + C_2^2}} \left[\left(1 - \frac{e^{-2\beta}}{L^2}\right) - \frac{E^2}{2m^2c^2} \left(\frac{L'}{L} - \beta'\right)^2 \right], \tag{45}$$

$$\Xi_2(u) = \frac{EC_2}{\sqrt{1 + C_2^2}} \left(1 - L e^{\beta}\right), \quad \Xi_3(u) = \frac{EU_3}{\sqrt{1 + C_2^2}}. \tag{46}$$

The formulae of two last subsections will simplify the next discussion.

4 DISCUSSION

The obtained exact solution of the evolutionary model demonstrates explicitly the following general properties.

(i) *Birefringence induced by curvature*

Using the formulae (40) with (30) or an explicit formula (42) we can conclude that the particle energy at the moment u depends on the initial data for the polarization four-vector. Since the polarization four-vector is normalized and orthogonal to the four-velocity (see (16)), only two initial components of this four-vector are independent (see (39)). In general case it is convenient to choose the E_2 and E_3 parameters to be independent ones. Since the particle energy depends on two initial polarization parameters, we deal with the bosonic analogue of the optical *birefringence*. Since the polarization parameters are involved into the particle energy formula just due to the tidal interaction, we can denote the effect of this type as birefringence induced by curvature[(6–8)]. We can compute the energy shift (41) for $E_2 = 0$ and for $E_3 = 0$. The corresponding ratio of the polarizationally induced energy shifts is equal to

$$[\mathcal{E} - mc^2]_{|E_2=0} : [\mathcal{E} - mc^2]_{|E_3=0} = (L' + L\beta')^2 : (L' - L\beta')^2. \tag{47}$$

Note that the polarizationally induced energy shifts for the first toy-model are of the same sign. In the weak GW field they differ only in the second order of β. Nevertheless, in the case of initially moving particle these shifts differ more considerably and have contributions linear in β.

The second toy-model demonstrates the degenerate case: if the E_2 and E_3 components vanish, i.e. the initial direction of the polarization three-vector coincides with that of the GW propagation, then the energy depends only on the polarization four-vector modulus, and birefringence is not displayed.

(ii) *Particle nongeodesic motion*

In the presence of the tidal interaction the particle motion is no longer the geodesic one. The tidal force in the right part of the formula (1) produces the particle *acceleration* and the *rotation* of the four-velocity vector. The formulae (30),(31),(26) describe explicitly these phenomena. Speaking about the particle rotation, we emphasize the following feature. If the initial direction of the particle motion is along the $0x^2$ axis (see the second toy - model), then in the field of tidal forces the U_2 component remains constant, but the $U_3(u)$ and $U_1(u)$ components appear. Certainly, it is not a standard rotation, because the instantaneous radius of the quasi-orbit grows. But one can find some analogy with the rotation of the charged particle in the nonhomogeneous magnetic field.

In principle, the phenomena of the particle nongeodesic acceleration and rotation are the supplementary contribution to the geodesic ones, caused by the minimal coupling with gravitation. This statement can be illustrated by the formula (45), in which the term in the parentheses describes the geodesic variation of the longitudinal velocity, and the last term appears due to the tidal interactions.

(iii) *Rotation of the polarization vector*

The effect of the rotation of the polarization vector is displayed by the formulae (29) and (26). The third two-dimensional matrix in the product in the right part of (26) is the standard rotation matrix, the $\psi(u)$ being the phase of rotation. Even if we shall neglect the term with right-dual Riemann curvature tensor in (30), nevertheless, the polarization four-vector (see (29)) will rotate due to the properties of the $X_\alpha(u)$ (26). In addition, the rotation of the particle three-velocity produces the supplementary rotation of the polarization three-vector, which is induced originally by curvature.

Thus, we can conclude that the gravitational waves effect on the vector boson can be considered as a combination of geodesic and tidal acceleration and rotation, leading to the birefringence phenomenon.

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